CHAPITRE 3 :

```
> restart;
> (2/(1+y)^3)*(x^2+y^2)*((a[1]*x+a[2]*y+a[3]*x*y+a[4]*x^2+a[5]*y^2
  )*(x+x*y)-(-y+x^2)*(b[1]*x+b[2]*y+b[3]*x*y+b[4]*x^2+b[5]*y^2))/(

  2*R*((x+x*y)*x-(-y+x^2)*y));
```

$$(x^2+y^2)\left((a_1 x+a_2 y+a_3 x y+a_4 x^2+a_5 y^2)(x+x y)-(-y+x^2)(b_1 x+b_2 y+b_3 x y+b_4 x^2+b_5 y^2)\right) \Big/ \left((1+y)^3 R((x+x y)x-(-y+x^2)y)\right)$$

```
> subs(x=rho*cos(phi),y=rho*sin(phi),%);
```

$$(\rho^2\cos(\phi)^2+\rho^2\sin(\phi)^2)($$
$$(a_1\rho\cos(\phi)+a_2\rho\sin(\phi)+a_3\rho^2\cos(\phi)\sin(\phi)+a_4\rho^2\cos(\phi)^2+a_5\rho^2\sin(\phi)^2)$$
$$(\rho\cos(\phi)+\rho^2\cos(\phi)\sin(\phi))-(-\rho\sin(\phi)+\rho^2\cos(\phi)^2)$$
$$(b_1\rho\cos(\phi)+b_2\rho\sin(\phi)+b_3\rho^2\cos(\phi)\sin(\phi)+b_4\rho^2\cos(\phi)^2+b_5\rho^2\sin(\phi)^2))\Big/($$
$$(1+\rho\sin(\phi))^3 R$$
$$((\rho\cos(\phi)+\rho^2\cos(\phi)\sin(\phi))\rho\cos(\phi)-(-\rho\sin(\phi)+\rho^2\cos(\phi)^2)\rho\sin(\phi)))$$

```
> f1:=simplify(subs(rho=R/(1-R*sin(phi)),%));
```

$$f1 := R(-b_2+b_4R^2\cos(\phi)^2+2b_1R\cos(\phi)+2b_2R\sin(\phi)+b_3R^2\cos(\phi)\sin(\phi)$$
$$-R\cos(\phi)^2 b_2\sin(\phi)+a_1\cos(\phi)^2R\sin(\phi)-a_3R\cos(\phi)^2\sin(\phi)-\sin(\phi)b_4R\cos(\phi)^2$$
$$-R^2b_2+R^2b_5-a_1\cos(\phi)^2-\cos(\phi)a_2\sin(\phi)-a_4R\cos(\phi)^3-\sin(\phi)b_1\cos(\phi)$$
$$-R\cos(\phi)^3b_1-R^2\sin(\phi)b_1\cos(\phi)+\cos(\phi)^3Rb_3-Ra_5\cos(\phi)+Ra_2\cos(\phi)$$
$$+\cos(\phi)^3Ra_5-\cos(\phi)^3Ra_2-Rb_3\cos(\phi)+\cos(\phi)^2R^2b_2-\cos(\phi)^2R^2b_5+\cos(\phi)^2b_2$$
$$-\sin(\phi)b_5R+\sin(\phi)\cos(\phi)^2b_5R)/(-1+R\sin(\phi))$$

```
> collect(numer(%),R);
```

$$(b_4\cos(\phi)^2+\cos(\phi)^2b_2+b_5-\sin(\phi)b_1\cos(\phi)+b_3\cos(\phi)\sin(\phi)-\cos(\phi)^2b_5-b_2)R^3+($$
$$-\cos(\phi)^2b_2\sin(\phi)+2b_1\cos(\phi)+2b_2\sin(\phi)-a_4\cos(\phi)^3-\cos(\phi)^3b_1-\sin(\phi)b_4\cos(\phi)^2$$
$$+\cos(\phi)^3b_3+a_1\cos(\phi)^2\sin(\phi)+\sin(\phi)\cos(\phi)^2b_5-\sin(\phi)b_5-a_5\cos(\phi)+a_2\cos(\phi)$$
$$+\cos(\phi)^3a_5-a_3\cos(\phi)^2\sin(\phi)-\cos(\phi)^3a_2-b_3\cos(\phi))R^2$$
$$+(-b_2+\cos(\phi)^2b_2-\cos(\phi)a_2\sin(\phi)-a_1\cos(\phi)^2-\sin(\phi)b_1\cos(\phi))R$$

```
> restart;
> g1:=Pi*R/2;
```

$$g1 := \frac{\pi R}{2}$$

```
> g11:=series(g1,R=0,8);
```

$$g11 := \frac{\pi}{2} R$$

```
> g2:=2+(Pi/R)-(Pi*R/2)+1/(2*R*(1-R^2)^(1/2))*(-2*Pi+2*Pi*R^2-(4-4
  *R^2)*arcsin(R));
```

$$g2 := 2 + \frac{\pi}{R} - \frac{\pi R}{2} + \frac{1}{2}\frac{-2\pi + 2\pi R^2 - (4 - 4R^2)\arcsin(R)}{R\sqrt{1 - R^2}}$$

```
> g22:=series(g2,R=0,8);
```

$$g22 := \frac{2}{3}R^2 + \frac{\pi}{8}R^3 + \frac{4}{15}R^4 + \frac{\pi}{16}R^5 + \frac{16}{105}R^6 + O(R^7)$$

```
> g3:=Pi*R/2-2*R^2;
```

$$g3 := \frac{1}{2}\pi R - 2R^2$$

```
> g33:=series(g3,R=0,8);
```

$$g33 := \frac{\pi}{2}R - 2R^2$$

```
> g4:=2+Pi/R-3*Pi*R/2-2*R^2+1/(2*R*(1-R^2)^(1/2))*(-2*Pi+4*Pi*R^2-
  2*Pi*R^4+(-4+8*R^2-4*R^4)*arcsin(R));
```

$$g4 := 2 + \frac{\pi}{R} - \frac{3\pi R}{2} - 2R^2 + \frac{1}{2}\frac{-2\pi + 4\pi R^2 - 2\pi R^4 + (-4 + 8R^2 - 4R^4)\arcsin(R)}{R\sqrt{1 - R^2}}$$

```
> g44:=series(g4,R=0,8);
```

$$g44 := \frac{2}{3}R^2 - \frac{3\pi}{8}R^3 - \frac{2}{5}R^4 - \frac{\pi}{16}R^5 - \frac{4}{35}R^6 + O(R^7)$$

```
> g6:=-2+Pi/R-Pi*R/2-2*(1-R^2)^(1/2)*arccos(R)/R;
```

$$g6 := -2 + \frac{\pi}{R} - \frac{\pi R}{2} - \frac{2\sqrt{1 - R^2}\arccos(R)}{R}$$

```
> g66:=series(g6,R=0,8);
```

$$g66 := -\frac{2}{3}R^2 + \frac{\pi}{8}R^3 - \frac{4}{15}R^4 + \frac{\pi}{16}R^5 - \frac{16}{105}R^6 + O(R^7)$$

```
> g8:=-2+Pi/R-3*Pi*R/2+2*R^2+2*(1-R^2)^(1/2)*arccos(R)*(R-1/R);
```

$$g8 := -2 + \frac{\pi}{R} - \frac{3\pi R}{2} + 2R^2 + 2\sqrt{1 - R^2}\arccos(R)\left(R - \frac{1}{R}\right)$$

```
> g88:=series(g8,R=0,8);
```

$$g88 := -\frac{2}{3}R^2 - \frac{3\pi}{8}R^3 + \frac{2}{5}R^4 - \frac{\pi}{16}R^5 + \frac{4}{35}R^6 + O(R^7)$$

```
> restart;
> (2/(1+2*y)^2)*(x^2+y^2)*((a[1]*x+a[2]*y+a[3]*x*y+a[4]*x^2+a[5]*y
  ^2)*(x+2*x*y)-(-y+x^2-y^2)*(b[1]*x+b[2]*y+b[3]*x*y+b[4]*x^2+b[5]

  *y^2))/(2*R*((x+2*x*y)*x-(-y+x^2-y^2)*y));
```

$$(x^2 + y^2)\left((a_1 x + a_2 y + a_3 x y + a_4 x^2 + a_5 y^2)(x + 2xy)\right.$$

$$-(-y+x^2-y^2)\,(b_1\,x+b_2\,y+b_3\,x\,y+b_4\,x^2+b_5\,y^2))\,/\,((1+2\,y)^2\,R$$

$$((x+2\,x\,y)\,x-(-y+x^2-y^2)\,y))$$

```
> subs(x=rho*cos(phi),y=rho*sin(phi),%);
```

$$(\rho^2\cos(\phi)^2+\rho^2\sin(\phi)^2)\,($$

$$(a_1\,\rho\cos(\phi)+a_2\,\rho\sin(\phi)+a_3\,\rho^2\cos(\phi)\sin(\phi)+a_4\,\rho^2\cos(\phi)^2+a_5\,\rho^2\sin(\phi)^2)$$

$$(\rho\cos(\phi)+2\,\rho^2\cos(\phi)\sin(\phi))-(-\rho\sin(\phi)+\rho^2\cos(\phi)^2-\rho^2\sin(\phi)^2)$$

$$(b_1\,\rho\cos(\phi)+b_2\,\rho\sin(\phi)+b_3\,\rho^2\cos(\phi)\sin(\phi)+b_4\,\rho^2\cos(\phi)^2+b_5\,\rho^2\sin(\phi)^2))\,/\,($$

$$(1+2\,\rho\sin(\phi))^2\,R$$

$$((\rho\cos(\phi)+2\,\rho^2\cos(\phi)\sin(\phi))\,\rho\cos(\phi)-(-\rho\sin(\phi)+\rho^2\cos(\phi)^2-\rho^2\sin(\phi)^2)\,\rho\sin(\phi)))$$

```
> f11:=simplify(subs(rho=R^2*sin(phi)+R*sqrt((R^2*sin(phi)^2+1)),%
  ));
```

$$f11 := R\,(96\,R^5\,a_4\cos(\phi)^5-R\,a_5\cos(\phi)+31\,R^3\cos(\phi)^3\,b_3$$

$$+72\cos(\phi)^2\,R^4\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}+18\,R^3\,a_4\cos(\phi)^5$$

$$-144\,R^6\cos(\phi)^4\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}-20\,R^4\,b_4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$+96\,R^6\,a_3\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}+80\,R^6\cos(\phi)^2\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$+32\,b_2\,R^4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,R^6\,a_3\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$-32\,a_3\cos(\phi)^2\,R^4\sqrt{1+R^2-R^2\cos(\phi)^2}-16\,R^6\,b_4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$+104\cos(\phi)^3\,R^5\,b_3+b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^2+80\,R^7\cos(\phi)^3\,b_3-18\,R^3\,a_5\cos(\phi)^5$$

$$-20\,R^4\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}-8\,b_2\,R^2\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,R^7\,a_5\cos(\phi)$$

$$+36\,R^3\,a_5\cos(\phi)^3+32\,R^7\,a_4\cos(\phi)^9-18\,R^3\,b_3\cos(\phi)^5-12\,b_1\cos(\phi)\,R^3$$

$$+128\,R^7\,a_5\cos(\phi)^7+112\,R^7\,b_3\cos(\phi)^7+5\,R\,a_2\cos(\phi)^3-144\,R^7\cos(\phi)^5\,b_3$$

$$+48\,R^5\,a_5\cos(\phi)^7-20\,a_2\cos(\phi)\,R^3+128\,R^7\,a_5\cos(\phi)^3-40\,R^5\cos(\phi)^5\,b_1$$

$$+16\,a_2\cos(\phi)^7\,R^5-18\,R^3\,a_5\cos(\phi)-8\,b_2\,R^4\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,R^7\,a_5\cos(\phi)^9$$

$$-32\,R^7\,a_4\cos(\phi)^3-16\,R^6\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}+96\,R^7\,a_4\cos(\phi)^5$$

$$-32\,R^7\cos(\phi)^8\sin(\phi)\,b_5-144\,R^7\cos(\phi)^4\sin(\phi)\,b_5+112\,R^7\cos(\phi)^6\sin(\phi)\,b_5$$

$$-124\,R^5\sin(\phi)\,b_5\cos(\phi)^4+48\,R^5\sin(\phi)\,b_5\cos(\phi)^6$$

$$-96\,R^6\,a_5\sin(\phi)\cos(\phi)^5\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$-16\,R^6\,b_3\cos(\phi)\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}+16\,b_2\,R^4\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$-96\,R^6\,a_3\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,R^6\cos(\phi)^8\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$-40\,b_2\,R^4\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}+32\,R^6\,a_3\cos(\phi)^8\sqrt{1+R^2-R^2\cos(\phi)^2}$$

$$+112\,R^6\cos(\phi)^6\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}-20\,R^3\cos(\phi)^5\,b_1-13\sin(\phi)\,R^3\,b_5$$

$$
\begin{aligned}
&- 4\,b_2 \sin(\phi)\,R - 12\,R^3 \sin(\phi)\,b_2 - 144\,R^5\,a_5 \cos(\phi)^5 - R \sin(\phi)\,b_5 - 96\,R^7\,a_4 \cos(\phi)^7 \\
&- R\,a_4 \cos(\phi)^3 - 48\,a_5\,R^5 \cos(\phi) - 5\,b_5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}\,R^2 - 192\,R^7\,a_5 \cos(\phi)^5 \\
&- 28\,R^5\,b_3 \cos(\phi) - 13\,R^3\,b_3 \cos(\phi) - 48\,a_2 \cos(\phi)^5\,R^5 + R\,b_3 \cos(\phi)^3 - 28\,R^5 \sin(\phi)\,b_5 \\
&- 16\,R^7 \sin(\phi)\,b_5 - 8\,b_2 \sin(\phi)\,R^5 + R\,a_5 \cos(\phi)^3 - 4\,R\,b_1 \cos(\phi) - R\,b_3 \cos(\phi) \\
&- 16\,a_2 \sin(\phi) \cos(\phi)\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&+ 32\,a_2 \sin(\phi) \cos(\phi)^3\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 20\,a_2 \cos(\phi)^5\,R^3 - 48\,R^5\,a_4 \cos(\phi)^7 \\
&- 6\,b_5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}\,R^2 \cos(\phi)^4 - 18 \sin(\phi)\,R^3\,b_5 \cos(\phi)^4 + 31 \sin(\phi)\,R^3\,b_5 \cos(\phi)^2 \\
&+ 20\,b_2\,R^2 \cos(\phi)^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 16\,R^7 \sin(\phi)\,b_4 \cos(\phi)^2 \\
&+ 104\,R^5 \sin(\phi)\,b_5 \cos(\phi)^2 + 80\,R^7 \cos(\phi)^2 \sin(\phi)\,b_5 + R \sin(\phi)\,b_5 \cos(\phi)^2 \\
&- 84 \cos(\phi)^4\,R^4\,b_5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 32 \cos(\phi)^6\,R^4\,b_5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 12\,b_2\,R^2 \cos(\phi)^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 96\,R^7\,a_3 \cos(\phi)^6 \sin(\phi) \\
&+ 64\,a_3 \cos(\phi)^4\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 96\,R^7\,a_3 \cos(\phi)^4 \sin(\phi) \\
&+ 64\,a_5 \sin(\phi)\,R^4 \cos(\phi)^3 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 32\,R^6\,a_5 \sin(\phi) \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 32\,R^7\,a_3 \cos(\phi)^8 \sin(\phi) \\
&- 32\,R^7\,a_3 \cos(\phi)^2 \sin(\phi) + 6\,R^2\,a_3 \cos(\phi)^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 16\,a_1 \cos(\phi)^6\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 124\,R^5\,b_3 \cos(\phi)^5 \\
&- 6\,a_5 \sin(\phi)\,R^2 \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 6\,a_5 \sin(\phi)\,R^2 \cos(\phi)^3 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 16\,a_2 \cos(\phi)\,R^5 - 32\,a_5 \sin(\phi)\,R^4 \cos(\phi)^5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 5 \sin(\phi)\,R^2\,b_3 \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 32\,a_3 \cos(\phi)^6\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 20 \sin(\phi)\,R^4\,b_3 \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 16\,a_2 \sin(\phi) \cos(\phi)^5\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 5\,R \cos(\phi)^3\,b_1 + 40\,a_2 \cos(\phi)^3\,R^3 \\
&+ 16\,R^5 \cos(\phi)^7\,b_1 + 32\,R^3 \cos(\phi)^3\,b_1 - a_1 \cos(\phi)^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 32\,b_1 \cos(\phi)^3\,R^5 \\
&- b_2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 48\,a_4 \cos(\phi)^3\,R^5 - 18\,R^3\,a_4 \cos(\phi)^3 \\
&+ 32\,R^6\,a_5 \sin(\phi) \cos(\phi)^7 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 32\,a_1 \cos(\phi)^4\,R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&- 32\,a_5 \sin(\phi)\,R^4 \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 5\,R^2\,b_4 \cos(\phi)^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&+ 11\,R^2 \cos(\phi)^2\,b_5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 48\,R^5\,b_3 \cos(\phi)^7 - 5\,R\,a_2 \cos(\phi) \\
&- 16\,R^7\,b_3 \cos(\phi) + 48\,a_2 \cos(\phi)^3\,R^5 - 12\,a_1 \cos(\phi)^2\,R^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} \\
&+ 52 \cos(\phi)^4\,R^4\,b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 20\,a_1 \cos(\phi)^2\,R^3 \sin(\phi) \\
&- 80\,R^6 \cos(\phi)^6\,b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 16\,a_1 \cos(\phi)^2\,R^5 \sin(\phi) - 80\,R^7 \cos(\phi)^6\,b_4 \sin(\phi)
\end{aligned}
$$

$$+ 64\, R^6 \cos(\phi)^4\, b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 76 \cos(\phi)^4\, R^5\, b_4 \sin(\phi) + 64\, R^7 \cos(\phi)^4\, b_4 \sin(\phi)$$
$$+ 32\, a_1 \cos(\phi)^4\, R^5 \sin(\phi) - 8\, b_1 \cos(\phi)\, R^2 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 12\, a_2 \sin(\phi) \cos(\phi)\, R^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 24 \sin(\phi)\, b_1 \cos(\phi)^3\, R^4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 32\, R^6\, a_4 \cos(\phi)^3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 64\, R^6\, a_4 \cos(\phi)^5 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 64\, R^6 \cos(\phi)^3\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 80\, R^6 \cos(\phi)^5\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 32\, a_4 \cos(\phi)^3\, R^4 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 52 \cos(\phi)^3\, R^4\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 8\, b_1 \cos(\phi)\, R^5$$
$$+ 96\, R^6\, a_5 \sin(\phi) \cos(\phi)^3 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 6 \cos(\phi)^4\, R^2\, b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 12\, a_1 \cos(\phi)^4\, R^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 5\, a_1 \cos(\phi)^2\, R \sin(\phi)$$
$$- 32\, R^4 \cos(\phi)^6\, b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - 48\, R^5 \cos(\phi)^6\, b_4 \sin(\phi)$$
$$+ 32\, R^6 \cos(\phi)^8\, b_4 \sqrt{1 + R^2 - R^2 \cos(\phi)^2} - b_1 \cos(\phi) \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- a_2 \sin(\phi) \cos(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 18 \cos(\phi)^4\, R^3\, b_4 \sin(\phi) + 20\, a_1 \cos(\phi)^4\, R^3 \sin(\phi)$$
$$+ 32\, R^7 \cos(\phi)^8\, b_4 \sin(\phi) - 18\, R^3\, a_3 \cos(\phi)^2 \sin(\phi) + 18\, R^3\, a_3 \cos(\phi)^4 \sin(\phi)$$
$$- 48\, R^5\, a_3 \cos(\phi)^2 \sin(\phi) + 96\, R^5\, a_3 \cos(\phi)^4 \sin(\phi) - 48\, R^5\, a_3 \cos(\phi)^6 \sin(\phi)$$
$$- 13\, R^3 \sin(\phi)\, b_4 \cos(\phi)^2 - 28\, R^5 \sin(\phi)\, b_4 \cos(\phi)^2 - R\, a_3 \cos(\phi)^2 \sin(\phi)$$
$$- R \sin(\phi)\, b_4 \cos(\phi)^2 - 16\, a_1 \cos(\phi)^6\, R^5 \sin(\phi) + 32\, R^3 \cos(\phi)^2\, b_2 \sin(\phi)$$
$$- 20\, R^3 \cos(\phi)^4\, b_2 \sin(\phi) + 32\, R^5 \cos(\phi)^2\, b_2 \sin(\phi) - 40\, R^5 \cos(\phi)^4\, b_2 \sin(\phi)$$
$$+ 16\, R^5 \cos(\phi)^6\, b_2 \sin(\phi) + 5\, R \cos(\phi)^2\, b_2 \sin(\phi)$$
$$+ 12\, a_2 \sin(\phi) \cos(\phi)^3\, R^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 12 \sin(\phi)\, b_1 \cos(\phi)^3\, R^2 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 32\, R^4\, a_4 \cos(\phi)^5 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 32\, R^6\, a_4 \cos(\phi)^7 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 16\, R^4 \sin(\phi)\, b_1 \cos(\phi)^5 \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 32\, R^4 \cos(\phi)^5\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$+ 32\, R^6 \cos(\phi)^7\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$
$$- 6\, a_4 \cos(\phi)^3\, R^2 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2} + 6 \cos(\phi)^3\, R^2\, b_3 \sin(\phi) \sqrt{1 + R^2 - R^2 \cos(\phi)^2}$$

$$
\begin{aligned}
&-16\,a_1\cos(\phi)^2 R^4\sqrt{1+R^2-R^2\cos(\phi)^2}-6\,R^2 a_3\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,R^7 b_3\cos(\phi)^9+144\,a_5 R^5\cos(\phi)^3-8\,b_1\cos(\phi)R^4\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\,)\,/\,(\\
&128\,R^5\sin(\phi)\cos(\phi)^2+96\cos(\phi)^2 R^6\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-48\,R^4\sqrt{1+R^2-R^2\cos(\phi)^2}\cos(\phi)^4+18\cos(\phi)^2 R^2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+32\,R^6\sqrt{1+R^2-R^2\cos(\phi)^2}\cos(\phi)^6+38\cos(\phi)^2 R^3\sin(\phi)-64\,R^5\sin(\phi)\cos(\phi)^4\\
&-96\,R^6\sqrt{1+R^2-R^2\cos(\phi)^2}\cos(\phi)^4+96\sin(\phi)R^7\cos(\phi)^2\\
&+96\cos(\phi)^2 R^4\sqrt{1+R^2-R^2\cos(\phi)^2}-96\sin(\phi)R^7\cos(\phi)^4+32\sin(\phi)R^7\cos(\phi)^6\\
&-32\,R^7\sin(\phi)-64\,R^5\sin(\phi)-6\,R\sin(\phi)-38\sin(\phi)R^3-48\,R^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,R^6\sqrt{1+R^2-R^2\cos(\phi)^2}-18\,R^2\sqrt{1+R^2-R^2\cos(\phi)^2}-\sqrt{1+R^2-R^2\cos(\phi)^2}\,)
\end{aligned}
$$

```
> collect(numer(%),R);
```

$$
\begin{aligned}
&(-16\sin(\phi)b_4\cos(\phi)^2+80\sin(\phi)b_5\cos(\phi)^2+112\sin(\phi)b_5\cos(\phi)^6-32\cos(\phi)^8\sin(\phi)b_5\\
&-16\sin(\phi)b_5-32\,a_4\cos(\phi)^3+128\,a_5\cos(\phi)^7-144\,b_3\cos(\phi)^5+32\,a_4\cos(\phi)^9\\
&+80\,b_3\cos(\phi)^3-32\,b_3\cos(\phi)^9+128\,a_5\cos(\phi)^3-32\,a_5\cos(\phi)-96\,a_4\cos(\phi)^7\\
&+112\,b_3\cos(\phi)^7-16\,b_3\cos(\phi)-192\,a_5\cos(\phi)^5+96\,a_4\cos(\phi)^5-32\,a_5\cos(\phi)^9\\
&+64\cos(\phi)^4 b_4\sin(\phi)-80\cos(\phi)^6 b_4\sin(\phi)+96\,a_3\cos(\phi)^4\sin(\phi)+32\,a_3\cos(\phi)^8\sin(\phi)\\
&-144\sin(\phi)b_5\cos(\phi)^4-96\,a_3\cos(\phi)^6\sin(\phi)+32\cos(\phi)^8 b_4\sin(\phi)-32\,a_3\cos(\phi)^2\sin(\phi)\\
&)R^8+(-32\,a_3\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}-16\,b_4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+64\cos(\phi)^4 b_4\sqrt{1+R^2-R^2\cos(\phi)^2}-80\cos(\phi)^6 b_4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+64\cos(\phi)^3 b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}-16\sin(\phi)b_3\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-80\cos(\phi)^5 b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}-16\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,a_4\cos(\phi)^7\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}+96\,a_3\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+64\,a_4\cos(\phi)^5\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}+32\cos(\phi)^7 b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-144\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\cos(\phi)^4-32\,a_4\cos(\phi)^3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+32\,a_5\sin(\phi)\cos(\phi)^7\sqrt{1+R^2-R^2\cos(\phi)^2}-96\,a_3\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+32\,a_3\cos(\phi)^8\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,a_5\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\cos(\phi)^8 b_5\sqrt{1+R^2-R^2\cos(\phi)^2}+32\cos(\phi)^8 b_4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+112\cos(\phi)^6 b_5\sqrt{1+R^2-R^2\cos(\phi)^2}+80\cos(\phi)^2 b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-96\,a_5\sin(\phi)\cos(\phi)^5\sqrt{1+R^2-R^2\cos(\phi)^2}+96\,a_5\sin(\phi)\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}\,)\\
&R^7+(-28\sin(\phi)b_4\cos(\phi)^2+104\sin(\phi)b_5\cos(\phi)^2+48\sin(\phi)b_5\cos(\phi)^6\\
&+32\cos(\phi)^2 b_2\sin(\phi)+16\cos(\phi)^6 b_2\sin(\phi)-16\,a_1\cos(\phi)^2\sin(\phi)+32\,a_1\cos(\phi)^4\sin(\phi)
\end{aligned}
$$

$$
\begin{aligned}
&-16\,a_1\cos(\phi)^6\sin(\phi)-40\cos(\phi)^4 b_2\sin(\phi)+48\,a_2\cos(\phi)^3+16\,a_2\cos(\phi)^7-16\,a_2\cos(\phi)\\
&-8\,b_1\cos(\phi)+16\cos(\phi)^7 b_1-48\,a_2\cos(\phi)^5-8\,b_2\sin(\phi)+32\cos(\phi)^3 b_1-40\cos(\phi)^5 b_1\\
&-28\sin(\phi)\,b_5-48\,a_4\cos(\phi)^3+48\,a_5\cos(\phi)^7-124\,b_3\cos(\phi)^5+104\,b_3\cos(\phi)^3\\
&+144\,a_5\cos(\phi)^3-48\,a_5\cos(\phi)-48\,a_4\cos(\phi)^7+48\,b_3\cos(\phi)^7-28\,b_3\cos(\phi)\\
&-144\,a_5\cos(\phi)^5+96\,a_4\cos(\phi)^5+76\cos(\phi)^4 b_4\sin(\phi)-48\cos(\phi)^6 b_4\sin(\phi)\\
&+96\,a_3\cos(\phi)^4\sin(\phi)-124\sin(\phi)\,b_5\cos(\phi)^4-48\,a_3\cos(\phi)^6\sin(\phi)-48\,a_3\cos(\phi)^2\sin(\phi)\\
&)R^6+(32\,b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^2+52\cos(\phi)^3 b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-16\,a_1\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}-8\,b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-16\sin(\phi)\,b_1\cos(\phi)^5\sqrt{1+R^2-R^2\cos(\phi)^2}+32\,a_2\sin(\phi)\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-16\,a_2\sin(\phi)\cos(\phi)^5\sqrt{1+R^2-R^2\cos(\phi)^2}-40\,b_2\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-16\,a_1\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}+64\,a_5\sin(\phi)\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+24\sin(\phi)\,b_1\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}+32\cos(\phi)^6 b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+72\cos(\phi)^2 b_5\sqrt{1+R^2-R^2\cos(\phi)^2}-32\,a_5\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,a_5\sin(\phi)\cos(\phi)^5\sqrt{1+R^2-R^2\cos(\phi)^2}+32\,a_4\cos(\phi)^5\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,a_4\cos(\phi)^3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}+64\,a_3\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-84\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^4-32\,a_3\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-20\sin(\phi)\,b_3\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}-32\cos(\phi)^5 b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-32\,a_3\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}-20\,b_4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+52\cos(\phi)^4 b_4\sqrt{1+R^2-R^2\cos(\phi)^2}-32\cos(\phi)^6 b_4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+32\,a_1\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}+16\,b_2\cos(\phi)^6\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-20\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}-8\,b_1\cos(\phi)\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&-16\,a_2\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\,)R^5+(-13\sin(\phi)\,b_4\cos(\phi)^2\\
&+31\sin(\phi)\,b_5\cos(\phi)^2+32\cos(\phi)^2 b_2\sin(\phi)-20\,a_1\cos(\phi)^2\sin(\phi)+20\,a_1\cos(\phi)^4\sin(\phi)\\
&-20\cos(\phi)^4 b_2\sin(\phi)+40\,a_2\cos(\phi)^3-20\,a_2\cos(\phi)-12\,b_1\cos(\phi)-20\,a_2\cos(\phi)^5\\
&-12\,b_2\sin(\phi)+32\cos(\phi)^3 b_1-20\cos(\phi)^5 b_1-13\sin(\phi)\,b_5-18\,a_4\cos(\phi)^3-18\,b_3\cos(\phi)^5\\
&+31\,b_3\cos(\phi)^3+36\,a_5\cos(\phi)^3-18\,a_5\cos(\phi)-13\,b_3\cos(\phi)-18\,a_5\cos(\phi)^5\\
&+18\,a_4\cos(\phi)^5+18\cos(\phi)^4 b_4\sin(\phi)+18\,a_3\cos(\phi)^4\sin(\phi)-18\sin(\phi)\,b_5\cos(\phi)^4\\
&-18\,a_3\cos(\phi)^2\sin(\phi))R^4+(-12\,a_2\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+6\,a_5\sin(\phi)\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}-12\,a_1\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}
\end{aligned}
$$

$$
\begin{aligned}
&- 6\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^4 - 5\sin(\phi)\,b_3\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- 6\,a_4\cos(\phi)^3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2} + 12\sin(\phi)\,b_1\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- 8\,b_1\cos(\phi)\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2} + 6\cos(\phi)^3\,b_3\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- 5\,b_4\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2} - 8\,b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+ 6\cos(\phi)^4\,b_4\sqrt{1+R^2-R^2\cos(\phi)^2} + 12\,a_1\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- 6\,a_5\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2} + 20\,b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^2\\
&+ 12\,a_2\sin(\phi)\cos(\phi)^3\sqrt{1+R^2-R^2\cos(\phi)^2} - 12\,b_2\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&+ 6\,a_3\cos(\phi)^4\sqrt{1+R^2-R^2\cos(\phi)^2} + 11\cos(\phi)^2\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- 6\,a_3\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2} - 5\,b_5\sqrt{1+R^2-R^2\cos(\phi)^2}\,)\,R^3 + (-5\,a_2\cos(\phi)\\
&+ a_5\cos(\phi)^3 - 5\,a_1\cos(\phi)^2\sin(\phi) - \sin(\phi)\,b_4\cos(\phi)^2 + 5\cos(\phi)^2\,b_2\sin(\phi) - a_4\cos(\phi)^3\\
&+ \sin(\phi)\,b_5\cos(\phi)^2 + b_3\cos(\phi)^3 + 5\,a_2\cos(\phi)^3 - 4\,b_2\sin(\phi) - a_5\cos(\phi) + 5\cos(\phi)^3\,b_1\\
&- b_3\cos(\phi) - \sin(\phi)\,b_5 - 4\,b_1\cos(\phi) - a_3\cos(\phi)^2\sin(\phi))\,R^2 + (\\
&b_2\sqrt{1+R^2-R^2\cos(\phi)^2}\,\cos(\phi)^2 - a_2\sin(\phi)\cos(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- b_2\sqrt{1+R^2-R^2\cos(\phi)^2} - b_1\cos(\phi)\sin(\phi)\sqrt{1+R^2-R^2\cos(\phi)^2}\\
&- a_1\cos(\phi)^2\sqrt{1+R^2-R^2\cos(\phi)^2}\,)\,R
\end{aligned}
$$

```
> restart;
```

```
> g[1]:=1+(Pi/2)*R-(1/R)*arctan(R)-R*arctan(R);
```

$$g_1 := 1 + \frac{\pi R}{2} - \frac{\arctan(R)}{R} - R\arctan(R)$$

```
> g[1,1]:=series(g[1],R=0,8);
```

$$g_{1,1} := \frac{\pi}{2}R - \frac{2}{3}R^2 + \frac{2}{15}R^4 - \frac{2}{35}R^6 + O(R^7)$$

```
> g[2]:=-1+(1/R)*arctan(R)+R*arctan(R);
```

$$g_2 := -1 + \frac{\arctan(R)}{R} + R\arctan(R)$$

```
> g[2,2]:=series(g[2],R=0,8);
```

$$g_{2,2} := \frac{2}{3}R^2 - \frac{2}{15}R^4 + \frac{2}{35}R^6 + O(R^7)$$

```
> g[3]:=-1+(Pi/2)*R-2*R^2+Pi*R^3+(arctan(R))*((1/R)-R-2*R^3);
```

$$g_3 := -1 + \frac{\pi R}{2} - 2R^2 + \pi R^3 + \arctan(R)\left(\frac{1}{R} - R - 2R^3\right)$$

```
> g[3,3]:=series(g[3],R=0,8);
```

$$g_{3,3} := \frac{\pi}{2}R - \frac{10}{3}R^2 + \pi R^3 - \frac{22}{15}R^4 + \frac{34}{105}R^6 + O(R^7)$$

```
> g[4]:=-1-R^2-(Pi/2)*R^3+(arctan(R))*((1/R)+2*R+R^3);
```

$$g_4 := -1 - R^2 - \frac{\pi R^3}{2} + \arctan(R)\left(\frac{1}{R} + 2R + R^3\right)$$

```
> g[4,4]:=series(g[4],R=0,8);
```

$$g_{4,4} := \frac{2}{3}R^2 - \frac{\pi}{2}R^3 + \frac{8}{15}R^4 - \frac{8}{105}R^6 + O(R^7)$$

```
> g[8]:=1+(Pi/2)*R+2*R^2+Pi*R^3+arctan(R)*((-1/R)+R+2*R^3);
```

$$g_8 := 1 + \frac{\pi R}{2} + 2R^2 + \pi R^3 + \arctan(R)\left(-\frac{1}{R} + R + 2R^3\right)$$

```
> g[8,8]:=series(g[8],R=0,8);
```

$$g_{8,8} := \frac{\pi}{2}R + \frac{10}{3}R^2 + \pi R^3 + \frac{22}{15}R^4 - \frac{34}{105}R^6 + O(R^7)$$